Equivalence of CFGs and PDAs

Lecture 22
Section 7.2

Robb T. Koether

Hampden-Sydney College

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Outline

1. Equivalence of PDAs and CFGs
   - Proof $\Rightarrow$
2. Example
3. Assignment

Outline

Equivalence of PDAs and CFGs

Theorem (Equivalence of PDAs and CFGs)

- *If G is a CFG, then there exists a PDA M such that $L(G) = L(M)$.*
- *If M is a PDA, then there exists a CFG G such that $L(M) = L(G)$.*

Outline

Proof ($\Rightarrow$).

- Given a CFG G, we will construct a PDA M such that $L(G) = L(M)$.
- Assume that G is in Chomsky Normal Form.
- Let M have three states:
  - $Q = \{q_0, q_1, q_2\}$.
  - q_0 is the start state.
  - q_2 is the accept state.

□

Equivalence of PDAs and CFGs

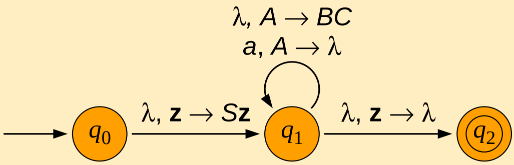
Proof (⇒).

- The transitions are
  - $\delta(q_0, \lambda, \mathbf{z}) = (q_1, S\mathbf{z})$
  - $\delta(q_1, \lambda, A) = (q_1, BC)$, for all productions $A \to BC$
  - $\delta(q_1, a, A) = (q_1, \lambda)$, for all productions $A \to a$.
  - $\delta(q_1, \lambda, \mathbf{z}) = (q_2, \lambda)$
- It is clear that $L(M) = L(G)$.

□

Equivalence of PDAs and CFGs

Proof ($\Rightarrow$).



For all productions $A \to BC$ and $A \to a$.

□

Outline

CFG to PDA

Example (CGG to PDA)

- Design a PDA for the context-free language with grammar

$$S \to SS \mid \mathbf{a}S\mathbf{b} \mid \mathbf{b}S\mathbf{a} \mid \lambda$$

- Process the input **aababb.**

Outline

Assignment

Assignment

- Section 7.2 Exercises 1, 2, 3, 9.